Exercise 40

Find the critical numbers of the function.

$$g(\theta) = 4\theta - \tan\theta$$

Solution

A critical number is a value of θ for which the derivative is zero or nonexistent. Take the derivative of the function.

$$g'(\theta) = \frac{d}{d\theta}(4\theta - \tan\theta)$$

$$= 4 - \sec^2\theta$$

Set $g'(\theta) = 0$ and solve for θ.

$$4 - \sec^2\theta = 0$$

$$\sec^2\theta = 4$$

$$\frac{1}{\cos^2\theta} = 4$$

The derivative does not exist if $\cos^2\theta = 0$.

$$\cos^2\theta = \frac{1}{4} \qquad\qquad\qquad\qquad\qquad \cos^2\theta = 0$$

$$\theta = \frac{2\pi}{3} + n\pi \quad \text{or} \quad \theta = \frac{\pi}{3} + n\pi \qquad\qquad \theta = \frac{\pi}{2} + n\pi, \quad n = 0, \pm 1, \pm 2, \dots$$